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SIMULATION OF POSITION ERRORS WHEN USING SELECTED ARMY MAP PROD--ETC(U)

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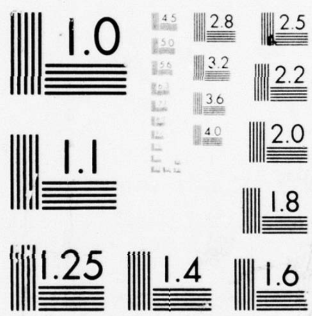

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Monterey, California

THESIS

SIMULATION OF POSITION ERRORS WHEN
USING SELECTED ARMY MAP PRODUCTS

by

Jimmy Wayne Cotner

December 1977

Thesis Advisors: S. H. Parry and P. A. W. Lewis

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by

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ABSTRACT

A simulation approach is given to estimating the distribution of navigational errors observed during a test of four Army map products, conducted by the US Army Combat Development Experimentation Command. Exponential and Gamma distributions are simulated with sample sizes identical to the actual data. The simulated samples are then replicated one thousand times to provide sample sizes of one thousand for key parameters such as skewness, standard deviation, coefficient of variation, range and selected quantiles. Comparison of these parameters is then made with the parameters observed in the data to determine the fit of simulated distributions.

TABLE OF CONTENTS

I. INTRODUCTION

Simulation to provide answers to difficult problems is not new. An attempt to simulate the distribution of navigational errors when using an existing or proposed map product is new. A field experiment conducted by the U. S. Army Combat Developments Experimentation Command (CDEC), MAPPRO III, during the period April-May 1977, was the motivation for a simulation approach to the distribution of navigational errors achieved during this experiment. This thesis is organized as follows to explain the simulation approach taken and the results of that simulation.

A. SECTION II - BACKGROUND

This section is important for the reader who is unfamiliar with the MAPPRO III experiment. It provides an overview of the experiment to include a discription of the experiments' location, players used, the map products used, design and objectives. The results of MAPPRO III and analysis technique CDEC utilized was provided as a departure point for a continuing analysis.

B. SECTION III - DESCRIPTION OF DATA

This section describes the data used for analysis. A description of the raw data, generation of the data points, transformations used, accuracy of the data, form for

analysis and quantity of the data is provided. A summary of the radial navigation errors is provided as a link to the goal of simulating these errors with an appropriate distribution.

C. SECTION IV - NATURE OF THE PROBLEM

The nature of the problem is a simple statement of the goals of the thesis in relation to what was accomplished by CDEC and what could be achieved by approximating the distribution of the radial navigation errors.

D. SECTION V - TESTS ON THE DATA

A description of the tests performed is given to determine pertinent facts about the data. Since a simulation of the radial navigation errors could be accomplished by simulation of the components of the radial error, tests were performed on the components to determine if these components were correlated. Since all of the components could not be definitely defined as uncorrelated, a component approach to simulation was eliminated from consideration and the remainder of the tests were performed to test the homogeneity of the data points of the radial errors.

E. SECTION VI - SIMULATED DISTRIBUTIONS

Two distributions were simulated to provide a close approximation to the distribution of the radial errors. The Exponential simulation was performed as a result of CDEC's

estimation that the distribution was Exponential. The second distribution simulated was the Gamma. A description of the method of simulation, the statistics investigated and compilation method is provided for both simulations. A comparison of the two simulated distributions and their associated parameters was then performed with comments made for each map product for daylight and for night navigation trials.

F. SECTION VII - CONCLUSIONS

The Gamma simulation provided the best approximation to the radial errors for all but one map product. Since CDEC could make no conclusion by use of radial errors on which map product performed best, use of the simulated distributions was used to discriminate between the maps. An alternative method of ranking the map products is described and included in this section. Guidelines for application of the result are also provided.

II. BACKGROUND

A. TEST OF NEW AND IMPROVED MAPS AND MAP PRODUCTS

1. General

During the period April-May 1977, The United States Army Combat Developments Experimentation Command (CDEC), Fort Ord, California conducted a Test of New and Improved Maps and Map Products (Short title MAPPRO III) in order to evaluate the four map products used by armor personnel in

(i) describing specific Military Graphic Information (MGI),

(ii) navigation of specified day and night routes to preselected checkpoints and

(iii) locating specified targets during daylight and night time environments.

2. Experiment Description

MAPPRO III was a three-part experiment. Part 1 evaluated the ability of selected armor personnel to identify MGI. Part 2 evaluated the effects on route planning, navigational accuracy and target location during daylight hours. Part 3 was a duplication of Part 2 except that it was conducted during periods of light levels less than 0.00025 foot candles.

3. Experiment Site

All trials and tests were conducted at Fort Hunter Liggett, California in the upper Gabilan Valley in flat-to-rolling and rolling-to-rough terrain.

4. Player/controller Participation

Thirty-six players participated in Part 1. Twelve Armor officers (2nd Lt) and thirteen armor mid-grade noncommissioned officers were available for participation in parts 2 and 3. However, only eight officers and eight noncommissioned officers actually participated in the trials of parts 2 and 3. A noncommissioned officer performed a dual role as vehicle driver and controller during all trials.

5. Map Products

Four 1:50,000 map products were involved in the tests:

(i) Standard Line (Map A). Army standard issue map.

(ii) Enriched Line (Map B). A map variety using the standard line map as a base. Militarily significant geographic information is added and utility enhanced through shaded relief.

(iii) Orthophotomap (Map C). A photo-based map product produced from a color mosaic of aerial photographs and printed using a three-color process.

(iv) Black and White Photo (Map D). A map produced from a single photograph of a mosaic of photographs and containing a military grid system and marginal information.

6. Experimental Design

Part 1 of the experiment was designed to test player personnel in their ability to identify MGI on the map products . This was accomplished through the use of several standard examinations prepared by the Engineer Topographic Laboratory. Parts 2 and 3 of the experiment were designed to evaluate route planning, navigational accuracy and target locations, accomplished by establishing four navigational lanes in the Gabilan Valley for both day and night trials. Along each lane were six preselected checkpoints and at the end of each lane, individual observation points (OP's) were used to locate targets. OP's at the end of lanes 1, 2 and 4 each had two targets to locate. The OP at the end of lane 3 had three targets to locate. The lanes, checkpoints, OP's and targets were different for day and night trials.

7. Experimental Objectives

The four principle objectives of the experiment were to obtain data on

(i) How clearly each map product depicts types of MGI during daylight conditions;

(ii) The usefulness of each map product in visualizing various terrain features and in answering relief-related questions;

(iii) of four different map products on route planning, navigational accuracy and target location during daylight hours;

(iv) The effects of four different map products on navigational accuracy and target location during periods of low light visability (0.00025 foot candles).

E. RESULTS OF MAPEFC III

1. Military Graphic Information (MGI)

The players answered written questions about MGI features most accurately with Map A and least accurately with Map D.

2. Day navigation

The Analysis of Variance (ANOVA) cross-products considering only the map/route combinations were statistically significant. A Chi-Square goodness of fit test of the sample distributions of day navigational radial error indicated that the closest fit was obtained by an exponential distribution.

3. Day Target Location

The largest radial error components of range and deflection occurred with Map D. There was little difference between these error components with the other maps.

4. Night Target Location

There was no statistically significant difference among map products. The dispersion of radial error was slightly larger with Map D.

5. Night Navigation

Of the ANOVA cross-products, the map by route and set of players by route were statistically significant. Navigation was least accurate with Map D and Map C on the most difficult route (Lane 4).

C. ANALYSIS TECHNIQUE

A three-way ANOVA was performed by CDEC on the navigation radial error. The ANOVA considered as possible sources of variation the map product, the routes and the sets of players. A set of players consisted of two officers and two noncommissioned officers who used all four map products on all four lanes. The players were assigned sets prior to record trials in a manner designed to minimize the differences between sets. This assignment was accomplished by reviewing players' scores on the written tests and

evaluating their field performance during explcratory trials. The best players were assigned tc work with the worst players, and the average players were grouped together. Rationale for establishing sets was that this permitted normal ANOVA techniques which require entries in all data cells. the raw data for the ANOVA's were converted by a log 10 transformation, which tends to normalize and stabilize the variance. One of the assumptions for this type of analysis is normality of data.

III. DESCRIPTION OF DATA

A. SOURCE

The raw data were furnished by the United States Army Combat Developments Experimentation Command, Fort Ord, California. The data points were generated during the conduct of the Test of New and Improved Maps and Map Products (Short title MAPPRO) Phase III, during the period 3 through 23 May 1977 at Fort Hunter Liggett, California.

B. NATURE OF THE DATA

1. Form

The raw data were in the form of ten-digit coordinates giving location to the nearest meter, codes identifying the lane, day/night trial, player, checkpoint and map product used.

2. Generation of Data Points

The data points were generated by daylight and night procedures. The daylight procedures were to assign a player to each of the four navigation lanes available, present the player with a map product with six designated checkpoints

for a player to plan his own navigation route to the identified checkpoints. For safety reasons during night trials, the players were provided the checkpoints and a designated route on the map product. As a player navigated his planned or designated route, his position was constantly monitored by electronic location designation devices. Once a player decided that he was at the designated checkpoint, he informed an his controller who activated a switch on his transmitting unit which accompanied the player and controller in their vehicle. The act of turning on the transmitter designated the location of the vehicle.

3. Transformation of the Data

The raw data points in the form of coordinates were then compared to the coordinates of the designated location. The actual location was then subtracted from the player's location which resulted in positive and negative values on an x,y axis with the designated location being (0,0) on the axis. Positive values are errors in the north and east directions. Negative values are errors in the south and west directions.

4. Accuracy

The accuracy of the radial errors calculated was determined by the measuring devices used. Because these calculations are to the nearest meter, the errors are probably continuous and identical values are due to rounding, even though the underlying distribution is continuous.

For player safety, the actual location of the player was continuously monitored. When a player deviated more than 500 meters during daylight trials, and more than 50 meters during night trials from his selected or prescribed

route, the player was questioned to determine if he knew his location. If he did not know his location, he was declared lost and returned to his previous checkpoint. This procedure, although assuring safety on the courses and being an operational necessity, limited the radial errors which would otherwise have been observed. The number of events in which a player was lost is an important parameter with an impact upon the performance of the four map products. The number of lost player events, however, was small in comparison to the number of navigation points used in the sample. Table I gives the number of lost player events for each map and the total lost player events. There were 96 checkpoints for each of the map products and 384 checkpoints total for each of the daylight and night trials. No record was made if a player recorded multiple lost player events when trying to reach a single checkpoint. Table I lists the number of lost player events recorded for each map during the two trials.

TABLE I - LOST PLAYER EVENTS

DAYLIGHT TRIALS

MAP A	MAP B	MAP C	MAP D	TOTAL
5	4	5	4	18

NIGHT TRIALS

MAP A	MAP B	MAP C	MAP D	TOTAL
1	2	5	5	13

The sample sizes for lost players is too small to make statistical inference in the data. The conclusions drawn by CEDC that there is no difference in the effect of the maps during Daylight Trials and that maps A and B performed

better with fewer lost player events during night trials was subjective and probably accurate. Without the benefit of knowing which radial errors were achieved after a lost player event and the possibility of purging those data points, nothing more can be said about these events and their effect was not considered in the remainder of the analysis.

5. Form for Analysis

The ten-digit coordinate of a player's location was split into two parts: the first five figures being East/West and the second five figures being North/South. These figures translate naturally into the x and y axis. Absolute miss distances from the actual checkpoints was then calculated by subtracting the North/South and East/West true checkpoint figures from the player's North/South and East/West figures. This provided X and Y readings which were individually squared and then the X-square was summed with the associated Y-square. A square-root was then taken of this sum and the result was the absolute miss distance from the actual checkpoint, subsequently referenced as the Radial Error and the data form used for analysis.

6. Quantity of Data

There were four map products to evaluate, four navigational lanes and six checkpoints on each lane. Each map product was used on each navigation lane resulting in 96 readings for each map during daylight trials and 96 readings for each map during night trials. The total number of readings was 384 for daylight trials and 384 for night trials.

C. SUMMARY OF THE RADIAL ERRORS

Summaries of the calculated parameters of the Radial Errors are given in Tables II and III. All values, except for Skewness and Kurtosis, are in meters. The Skewness is a measure of the symetry about the mean. The Kurtosis indicates the shape of the density function with large values indicating sharp peak and small values indicating a flat peak.

TABLE II - SUMMARY OF RADIAL ERRORS - DAYLIGHT TRIALS

	MAP A	MAP B	MAP C	MAP D
Mean	42.14	44.24	47.23	47.17
Standard Deviation	63.60	65.18	61.06	60.24
Skewness	2.516	2.687	2.062	1.831
Kurtosis	5.900	7.802	3.654	2.963
10th Quantile	3.606	3.606	5.385	3.162
25th Quantile	8.395	7.176	9.641	7.839
50th Quantile	19.35	17.73	22.34	17.46
75th Quantile	38.97	54.15	51.92	72.01
90th Quantile	115.8	107.8	138.7	126.6
Maximum	319.7	357.8	274.6	282.0

TABLE III - SUMMARY OF RADIAL ERRORS - NIGHT TRIALS

	MAP A	MAP B	MAP C	MAP D
Mean	46.32	63.08	54.60	63.00
Standard Deviation	60.20	69.78	74.75	60.19
Skewness	2.268	2.773	3.818	1.490
Kurtosis	5.283	11.33	20.63	2.253
10th Quantile	3.162	5.385	5.385	4.243
25th Quantile	7.567	12.67	10.79	14.49
50th Quantile	19.26	45.79	33.58	50.26
75th Quantile	61.48	87.23	62.60	86.79
90th Quantile	109.0	137.4	135.1	161.0
Maximum	299.0	464.1	558.4	305.1

IV. NATURE OF THE PROBLEM

CDEC utilized a transformation of the radial errors in order to perform ANOVA on the data to determine differences in the map products. The only attempt to identify the distribution of radial errors was a Chi-Square Goodness of Fit Test which accepted the hypothesis that the errors were Exponentially distributed. Since CDEC could not rank the effectiveness of the map products using ANOVA techniques, knowledge of the distribution of radial errors could provide better methods to arrive at a decision as to which map product performed best. The problem, therefore, is to determine the actual distribution of the radial errors. This Thesis is a simulation approach to finding a distribution which best fits the actual distribution of the navigation radial errors. Two distributions, Exponential and Gamma, were simulated and compared to the actual data. Once a distribution for the radial errors has been approximated, decisions about the performance of each map product can be made.

V. TESTS ON THE DATA

A. PARAMETRIC TESTS

One approach to simulating the radial errors would be to simulate the X and Y errors and then convert these values to radial errors. In order to simulate the X and Y values, knowledge of the correlation between the X and Y values was required. If little or no correlation existed between these values and the distribution of these values could be approximated by the Normal distribution, the problem of simulation would be reduced. An analysis of the X and Y values using a histogram was not encouraging in that the values did not appear Normally distributed. The following test assumes that the X and Y values are Normally distributed. The BIMED Statistical Package [Ref. 6] was used in the following test.

1. Bivariate Plotting

The bivariate plotting of the X, Y data was performed to observe the relative locations of the (X,Y) errors in relation to a common checkpoint and to determine if trends could be observed. These plots were prepared for each map for daylight and night trials. With each plot, the BIMED Statistical Package provided the correlation, the means and standard deviations of X and Y, a linear regression line and the residual mean squares. The results of this analysis are described in the following sections.

2. Correlation Between X and Y

Since the BIMED bivariate plotting of the X and Y is based on the assumption of Normality, tests on the Coefficient of Correlation were based on the hypothesis that the X and Y values were independent if the absolute value of the Coefficient of Correlation was not too large. A Student's "t" test was used for the hypothesis that the Coefficient of Correlation equaled zero. The "t" statistic was calculated by:

Let: r = Sample Coefficient of Correlation

N = 96

Then: t = r(SQRT(N - 2))/SQRT(1 - r*r)

The significance of each Coefficient of Correlation determines whether in fact the X and Y values are independent (for Normally distributed X and Y values). The results for daylight navigation trials indicated a negative coefficient of correlation of 0.282 (Significant at Alpha = 0.003) for Map A, a positive coefficient of correlation of 0.154 (Significant at Alpha = 0.07) for Map B, a negative coefficient of correlation of 0.223 (Significant at Alpha = 0.008) for Map C and a positive coefficient of correlation of 0.085 (Significant at Alpha = 0.20) for Map D. Map D accepts the hypothesis for any Alpha less than 0.20. Map B accepts the hypothesis for any Alpha less than 0.07. Maps A and C reject the hypothesis for reasonable Alpha values. This indicates that the values for at least two of the maps are correlated under the assumption of Normality. The results for night navigation trials all reject the hypothesis.

3. Observed Trends in the Plots

For the daylight trials, most of the (X,Y) errors were to the West and South of the actual checkpoint. This result is not surprising, given the human tendency to drift to the right when attempting to go in a straight line, and most of the routes to the checkpoints followed a general asmuthe of East to Southeast. During the night trials, the same trend in West and South errors was observed. Maps A and C show trends in that the radial errors are in a 250 meter band from Northwest to Southeast. Map B and D do not exhibit this trend.

B. NONPARAMETRIC CORRELATION TESTS ON X AND Y DATA

To further investigate the possibility of correlation between the X and Y values, several Nonparametric Tests were performed. The bivariate Plotting and resultant correlation provided previously, assume that the X and Y values are Normally distributed. Since this fact has not been shown, Nonparametric tests for correlation were performed. All tests shown below were performed by the BIMED Statistical Package.

1. Spearman Rank Correlation Coefficient (Spearman Rho)

The results of the test were as follows:

	MAP A	MAP B	MAP C	MAP D
Daylight Trials	-0.275	-0.015	-0.069	0.007
Night Trials	-0.209	-0.324	-0.410	-0.272

Since the number of samples is greater than 30, the distribution of the Spearman Rho is adequately approximated, under the hypothesis of no trend, by a normal distribution with mean zero and variance (V) equal to the recipical of the sample size minus one (V = 1/(96 - 1)). The value 0.01053 was then used as the variance, so the Standard Deviation was 0.1026. The hypothesis of no trend could then be rejected with an Alpha = 0.10, if the absolute value of Spearman Rho was greater than 0.13150. By this criteria Map A, during daylight trials, and all maps during the night trials, show correlation between the X and Y values.

2. Other Correlation Tests

The Kendall Rank Correlation Coefficient Test was also performed and supported the findings of the Spearman Rho and is not included. Since a correlation is indicated between the X and Y values, a simple simulation of the X and Y values was not possible and the approach was eliminated from consideration.

C. NONPARAMETRIC TESTS FOR HOMOGENEITY OF RADIAL ERRORS

Since the X and Y values were not uncorrelated for all maps, the simulation of the radial errors by simulating X and Y was discarded and emphasis was placed on the radial errors. Tests were performed on the radial error values to determine if the underlying distributions for the radial errors from all map products were the same. Two Nonparametric tests were used: The Kolmogorov-Smirnov Test is a pair-wise test and The Kruskall-Wallis Test is a multi-sample test to determine if the underlying distributions are the same.

1. Kruskall-Wallis Test on Radial Errors

A Kruskall-Wallis Test was performed on the data to determine if the values were drawn from the same distribution. statistic (H) is generated by merging all of the sample data, rank ordering the data and then grouping the data into their original sets and summing the ranks for each set. The Kruskall-Wallis statistic, H, is given by

$$H = (12/N(N + 1))(R1/n + R2/m + R3/l + R4/p) - 3(N + 1)$$

Where:

$R1$=Sum of ranks for Map A

$R2$=Sum of ranks for Map B

$R3$=Sum of ranks for Map C

$R4$=Sum of ranks for Map D

$n = m = l = p = 96$

$N = n + m + l + p = 384$

The distribution of H is then Chi-square with three degrees of freedom.

Comparisons of the radial errors for the four maps for daylight and night navigation trials could not reject the hypothesis that the radial errors for daylight or the radial errors for night came from the same underlying distribution.

2. Kolmogorov-Smirnov Test of Radial Errors

a. Daylight Navigation Trials

Emperical Cummulative Distribution Functions were calculated for each map and plotted on the same scale. This was accomplished by incrementing by five meters, counting the number of values which were equal to or less than that increment sum and dividing by the sample size of ninety-six. Using pair-wise calculations and an Alpha value of 0.10, the results were as follows.

Let $d = \text{Max}\,|F1(X) - F2(X)|$

Accept at Alpha=0.10 if d <1.22(Sqrt(2/96))=0.17609

Map A vs Map B, d=0.06<0.17609

Map A vs Map C, d=0.06<0.17609

Map A vs Map D, d=0.14<0.17609

Map B vs Map C, d=0.06<0.17609

Map B vs Map D, d=0.14<0.17609

Map C vs Map D, d=0.12<0.17609

The Kolmogorov-Smirnov Test cannot reject the hypothesis that the underlying distributions are the same.

VI. SIMULATED DISTRIBUTIONS

After observing the histograms of the actual radial errors and recognizing the fact that the radial errors would be positive values, the simulation of random variables which had values less than zero were eliminated from consideration. The CDEC report used a Chi-Square Goodness of Fit Test and estimated the distribution of errors to be Exponential and was, therefore, included in the investigation. The Gamma distribution was investigated to try to explain the shape of the histograms and the larger number of values near zero. A lack of time prevented the investigation of possible combinations of distributions.

A. SIMULATION OF THE EXPONENTIAL DISTRIBUTION

1. General

As stated in Chapter II, CDEC performed Chi-Square Goodness of Fit tests to the radial errors and stated that the exponential distribution provided the best fit. Since the Chi-Square Test is sensitive to how the data was sectioned and grouped at the tails of the distributions, this test is not very suitable.

2. Histogram and Statistics Investigated

By reading the radial errors into the HISTF/g Library package, a histogram of the data was produced and twenty-six pertinent statistics were generated. The mean, coefficient of skewness, and coefficient of kurtosis are three of the twenty-six statistics and were computed as shown below.

Mean= X(i)/N

Skewness=(third Central Moment)/(Standard Devation)

Kurtosis=(Fourth Central Moment)/(Standard Devation) - 3

The skewness and kurtosis of distributions are of interest because they have ranges of values for particular distributions. A range of values was obtained to compare with the real data by simulating the occurence of the random variable over many replications.

3. Method of Simulation

The simulation procedure used was to calculate the estimate of the mean for actual radial errors and to use this estimate as the parameter for an Exponential random variate generator. The random variate generator is described below

X=Random Variate

U = Uniform (0,1) random number

M=Estimate of the Mean

$$X=-(M)\ln(U)$$

Ninety-six of these Exponential (X) variates were generated to simulate the sample size of real navigation radial errors. These generated samples were then replicated one thousand times.

4. Compilation of Replications

Each sample of size 96 was read into a modified HISTF/G subroutine in order to generate the sample statistics. The one thousand replications provided arrays for each statistic of size one thousand. The arrays for the mean, coefficient of skewness, coefficient of Kurtosis, Coefficient of Variation, Median, 50th Quantile, 75th Quantile and 90th Quantile were then put into the HISTF/G Libarary program. The output provided a sample distribution of the statistics which were generated by an Exponential(M) distribution through which the actual data were compared. The tables of these results are in Appendix E.

B. SIMULATION OF THE GAMMA DISTRIBUTION

1. Computation of Parameters

By inspecting the histogram generated by the experimental radial errors, the Gamma distribution appeared a logical possibility. To compute the scale and location parameters for the Gamma distribution, the calculated mean and variance were used as follows:

Mean=Shape*Scale

Variance =Shape*(Scale)**2

This resulted in Alpha(Shape) and Beta(Scale) parameters calculated as follows:

Beta=Variance/Mean

Alpha=Mean/Beta

The calculated parameters of the distribution are given in Table IV.

TABLE IV - PARAMETERS OF GAMMA DISTRIBUTION

	DAYLIGHT TRIALS	NIGHT TRIALS
Map A:	Alpha=0.43901	Alpha=0.59214
	Beta=95.99487	Beta=78.22957
Map B:	Alpha=0.46086	Alpha=0.81735
	Beta=96.00516	Beta=77.17915
Map C:	Alpha=0.59832	Alpha=0.53357
	Beta=78.94075	Beta=102.33755
Map D:	Alpha=0.61320	Alpha=1.09553
	Beta=76.92229	Beta=57.50502

2. Method of Gamma Simulation

In order to generate sample sizes of ninety-six and replicate these samples one thousand times, a rejection method was utilized to generate Gamma variates. The Gamma random variate generator used was from the Directory of Routines in the International Mathematical and Statistical

Library (IMSL) at the W. R. Church Computer Center, Naval Postgraduate School, Monterey, California. Since all IMSL routines are proprietary, the program cannot be included. Each of the samples of size ninety-six were then used as entries into the Modified HISTG/F Subroutine to generate sample sizes of one thousand for the twenty-six parameters for both the Daylight and Night Navigation errors.

3. Compilation of Replications

The compilation procedure was identical to that used by the Exponential simulation. Results of the compilation are in Appendix C.

C. COMPARISON OF THE TWO SIMULATED DISTRIBUTIONS TO THE ACTUAL DATA

The comparisons below are made between the four map products for the parameters given. Replications of the parameters provided Normally distributed data for all but the Kurtosis and the Range. The support for a particular distribution was chosen by letting Alpha equal 0.10 and performing the standard Normal Test. Replications of the simulated distributions did not provide Normally distributed Kurtosis or Range, so standard application of statistical techniques was not appropriate. All Kurtosis and Range parameters were checked to insure feasibility and not used to discriminate between the maps. Since both parameters for generating simulated Exponential the means of the distributions are not used as a discriminating parameter. The distribution statistics for the exponential and gamma distributions presented in tables V through XII are coded as follows:

AV - The Actual Value.

EMN - The Exponentially Generated Mean of the Parameter.

ESD - The Standard Deviation of the Exp Generated Mean.

GMN - The Gamma Generated Mean of the Parameter.

GSD - The Standard Deviation of the Gamma Generated Mean.

TABLE V - DISTRIBUTION STATISTICS - DAYLIGHT TRIALS - MAP A

Parameter	AV	EMN	ESD	GMN	GSD
Mean	42.14	42.23	4.233	41.87	6.523
Std Dev	63.60	41.91	5.814	62.29	12.23
Coeff Var	1.509	0.993	0.101	1.488	0.175
Range	318.7	217.2	53.66	347.7	109.2
Skewness	2.516	1.823	0.558	2.632	0.782
Kurtosis	4.257	3.732	8.727	6.740	6.815
50th Qnt	19.35	29.55	4.376	17.24	4.432
75th Qnt	38.97	58.58	7.563	54.23	10.98
90th Qnt	115.8	93.20	12.03	110.7	21.44

The generated parameters for Map A shown in Table V, support the Gamma distribution of the radial errors for this map. The Standard Deviation, Coefficient of Variation and all quantiles consistently reject the Exponential distribution and accept the Gamma distribution.

TABLE VI - DISTRIBUTION STATISTICS - DAYLIGHT TRIALS - MAP B

Parameter	AV	EMN	ESD	GMN	GSD
Mean	44.25	44.17	4.637	41.87	6.523
Std Dev	65.18	43.71	6.441	63.97	12.24
Coeff Var	1.473	0.989	0.098	1.448	0.178
Range	356.8	227.1	58.11	357.7	116.5
Skewness	2.687	1.809	0.572	2.632	0.782
Kurtosis	7.802	4.237	3.949	8.727	6.740
50th Qnt	17.73	30.96	4.420	19.12	4.795
75th Qnt	54.15	61.70	7.941	54.23	10.98
90th Qnt	107.8	97.74	13.28	110.7	21.44

The generated parameters for Map B, shown in Table VI, again strongly support the Gamma distribution of the radial errors for this map. The Standard Deviation, Coefficient of Variation and 50th Quantile support a Gamma distribution. The rest of the parameters support either Gamma or Exponential distribution. The Skewness actual value is within a normal range for both generated Exponential and Gamma. For the Exponential, the actual value is 1.535 Standard Deviations from the generated mean. For the Gamma, the actual value is 0.070 Standard Deviations from the generated mean. Clearly, the actual value is more "comfortable" in the Gamma distribution, but for any reasonable Alpha value (<0.10) the hypothesis that the actual value is from the Exponential distribution cannot be rejected. The 75th and 90th Quantiles equally support either a Gamma or Exponential distribution.

TABLE VII - DISTRIBUTION STATISTICS - DAYLIGHT TRIALS - MAP C

Parameter	AV	EMN	ESD	GMN	GSD
Mean	47.23	47.40	4.938	47.45	6.146
Std Dev	61.06	46.87	6.666	59.93	10.26
Coeff Var	1.293	0.988	0.095	1.262	0.129
Range	273.6	241.7	57.54	318.2	108.4
Skewness	2.062	1.782	0.506	2.216	0.618
Kurtosis	3.654	3.967	3.278	6.106	4.747
50th Qnt	22.34	33.24	4.902	25.42	5.193
75th Qnt	51.92	66.03	8.254	64.76	10.18
90th Qnt	138.7	105.1	14.09	118.4	19.54

The generated parameters for Map C, shown in Table VII, support the Gamma distribution of the radial errors for this map. The Standard Deviation, Coefficient of Variation, 50th Quantile and 90th Quantile support the Gamma distribution. The Skewness and 75th Quantile support either Exponential or Gamma distribution. For the Skewness, the actual value is 0.553 Standard Deviations from the Exponential mean and 0.233 Standard Deviations from the Gamma mean. For the 75th Quantile, the actual value is 1.709 Standard Deviations from the Exponential mean and 1.261 Standard Deviations from the Gamma mean.

TABLE VIII - DISTRIBUTION STATISTICS - DAYLIGHT TRIALS - MAP D

Parameter	AV	EMN	ESD	GMN	GSD
Mean	47.17	47.22	4.651	47.23	6.381
Std Dev	60.24	46.51	6.537	59.35	10.18
Coeff Var	1.277	0.985	0.095	1.256	0.131
Range	281.0	239.9	59.55	318.4	88.99
Skewness	1.831	1.787	0.534	2.256	0.656
Kurtosis	2.963	4.050	3.482	6.438	5.027
50th Qnt	17.46	33.12	4.751	25.67	5.155
75th Qnt	72.01	65.74	8.081	64.54	10.66
90th Qnt	126.6	104.4	12.99	116.7	19.60

The generated parameters for Map D, shown in Table VIII, support the Gamma distribution of the radial errors for this map. The Standard Deviation, Coefficient of Variation and 50th Quantile support the Gamma distribution. The remaining parameters support either a Gamma or Exponential distribution. Except for the Skewness, the Gamma distribution fit is better than the Exponential distribution. The Exponential simulation provides a better fit of the actual Skewness.

TABLE IX - DISTRIBUTION STATISTICS - NIGHT TRIALS - MAP A

Parameter	AV	EMN	ESD	GMN	GSD
Mean	46.32	46.42	4.653	46.33	5.944
Std Dev	60.20	46.06	6.391	59.65	1.018
Coeff Var	1.300	0.993	0.100	1.287	0.141
Range	298.0	238.7	59.81	322.2	92.13
Skewness	2.268	1.832	0.558	2.322	0.689
Kurtosis	5.283	4.257	3.732	6.860	5.530
50th Qnt	19.26	32.48	4.810	24.50	4.929
75th Qnt	61.48	64.39	8.313	62.78	10.37
90th Qnt	109.0	102.4	13.23	115.6	18.63

The generated parameters for Map A, shown in Table IX, support the Gamma distribution of the radial errors for this map. The Standard Deviation, Coefficient of variation and 50th Quantile support the Gamma distribution. All of the remaining generated parameters support either the Exponential or Gamma distribution.

TABLE X - DISTRIBUTION STATISTICS - NIGHT TRIALS - MAP B

Parameter	AV	EMN	ESD	GMN	GSD
Mean	63.08	62.98	6.612	62.97	7.138
Std Dev	69.78	62.33	9.184	68.62	10.47
Coeff Var	1.106	0.989	0.098	1.088	0.114
Range	463.1	323.8	82.85	360.4	96.13
Skewness	2.773	1.809	0.572	2.322	0.689
Kurtosis	11.33	4.237	3.949	5.074	4.629
50th Qnt	45.79	44.15	6.302	40.62	6.758
75th Qnt	87.23	87.98	11.32	87.77	12.12
90th Qnt	134.4	139.4	18.93	145.9	21.03

The generated parameters for Map B, shown in Table X, support either the Gamma or Exponential distribution of the radial errors for this map. No statistical discriminator can be found to determine which distribution is correct. This fact is not surprising, since the shape parameter for the Gamma distribution was 0.817, close to the shape parameter for the Gamma to become an Exponential (Shape = 1.000).

TABLE XI - DISTRIBUTION STATISTICS - NIGHT TRIALS - MAP C

Parameter	AV	EMN	ESD	GMN	GSD
Mean	54.60	54.79	5.708	55.15	7.427
Std Dev	74.75	54.18	7.706	73.90	13.04
Coeff Var	1.369	0.989	0.095	1.340	0.151
Range	557.4	279.4	66.41	397.8	117.6
Skewness	3.818	1.782	0.572	2.352	0.699
Kurtosis	3.654	3.976	3.278	6.106	4.747
50th Qnt	33.58	38.42	5.667	27.01	5.950
75th Qnt	62.60	76.33	9.542	74.20	12.76
90th Qnt	135.1	121.5	16.29	141.0	24.61

The generated parameters for Map C, shown in Table XI, support the Gamma distribution of the radial errors for this map. The Standard Deviation, Coefficient of Variation and Skewness support the Gamma distribution. The Quartiles support either an Exponential or Gamma distribution.

TABLE XII - DISTRIBUTION STATISTICS - NIGHT TRIALS - MAP D

Parameter	AV	EMN	ESD	GMN	GSD
Mean	63.00	63.07	6.212	40.20	3.982
Std Dev	60.19	62.12	8.731	37.92	5.153
Coeff Var	0.985	0.985	0.095	0.944	0.090
Range	302.3	320.5	79.53	194.2	44.55
Skewness	1.490	1.787	0.534	1.698	0.507
Kurtosis	2.253	4.050	3.482	3.652	3.214
50th Qnt	50.26	44.23	6.346	29.14	4.069
75th Qnt	86.79	87.81	10.79	56.11	6.752
90th Qnt	161.0	139.5	17.35	87.18	11.22

The generated parameters for Map D, shown in Table XII, support the Exponential distribution of radial errors for this map. The Standard Deviation and all quantiles support the Exponential distribution. Only the Coefficient of Variation and Skewness support either the Gamma or Exponential distribution. The resulting support for the Exponential distribution is not unexpected given the mean and standard deviation relationship in the actual values.

VII. CONCLUSIONS

A. THE DISTRIBUTION OF RADIAL ERRORS

The simulated gamma distribution provided the best fit of the parameters to the radial errors. Since the Exponential distribution is a special case of the Gamma distribution, it was not surprising that the Exponential distribution provided good fits to the radial errors when CDEC performed the Chi-Square Goodness of Fit Test. Map D, night navigation trials, is probably best approximated by the Exponential distribution. For all other trials and maps, the errors are best approximated by the Gamma distribution.

B. COMPARISON OF THE MAPS BY QUANTILES

Since no statistical difference could be detected between the means of the maps, a comparison of the quantiles was made. The purpose of the field experiment was to determine effectiveness of current and proposed maps, thus the mean is a useful, but not an overly important statistic to observe. The real goal would seem to be to find a map which resulted in the largest percentage of the errors closest to the checkpoint. the 75th and 90th Quantiles are important Therefore, statistics.

1. The 75th Quantile

Map A's real quantiles were statistically better than the other three maps during the daylight trials and better than all but Map D during the night trials. Thus, when Map A was used, 75 percent of the values were significantly better than the other maps.

2. The 90th Quantile

No statistical difference exists between the maps at the 90th quartile during the daylight trials. The night trials produced a difference only in that Map D was the statistically worst map. Based upon this statistic, Map D was eliminated as the worst alternative among the four map products.

C. COMPARISON OF THE MAPS BY AREAS OF THE ERRORS

An alternative method of ranking map products involves returning to the X and Y values. Because of the method used to generate the X and Y values, a common checkpoint was designated and the navigation errors can be placed on a single axis for each map product and trial. Observation of these errors on a common plot leads to another method of ranking. Since the goal of MAPPRO III was to determine the performance of the map products, one measure could be to rank the maps by the size of the areas generated by a percentage of the results. Remembering that positive and negative values were available for X and Y values, the lower percentiles would , in general, reflect negative values and the higher percentiles would reflect positive values. By

calculating the absolute distance between lower and higher percentiles, a percentage of values will fall into that absolute distance. By doing this for both X and Y values and then multiplying the absolute distance for X by the absolute values for Y, an area which includes an approximation to a percentage of the (X,Y) values in that area results. The size of the area then becomes an analysis tool to rank the maps. An analogy exists with the percentiles given in previous sections for radial errors. Table XIII provides the results of calculating the areas of 50-percent (75th Percentile minus the 25th Percentile) and 80-percent (90th Percentile minus the 10th Percentile) areas. The figures are in meters squared and the rank is given with the smallest area being ranked 1 and the largest area being ranked 4. Fortunately, differences did not occur in the rankings for 50-percent and 80-percent. Map A ranks first for both day and night trials as it did using radial errors.

TABLE XIII - RANKINGS OF MAPS BY AREAS

MAP	50PCT	80PCT	RANK
DAYLIGHT TRIALS			
A	304	1754	1
B	342	6298	2
C	407	7735	4
D	407	7735	3
NIGHT TRIALS			
A	708	8284	1
B	2365	13640	3
C	1300	9477	2
D	2336	15300	4

The ANOVA technique used by CDEC was appropriate for the distribution of the error data. However, a ranking of the maps during navigational trials was not possible. Through the other techniques employed, a ranking is possible for the navigation trials. Map A, the map currently in use, is the best of the alternatives presented.

D. APPLICATION OF THE RESULTS

The sample size for the MAPPFQ III experiment was not large enough to reach a definite conclusion on the data. With the data available, the conclusion that the radial errors experienced during navigational trials is Gamma distributed is valid. This knowledge can be used during

future (and past) field experiments with map products to perform parametric tests on the estimated parameters generated by the actual data. The parameters for the Gamma distribution should be calculated as shown to provide the best results. Use of the Fortran routines supplied will permit the simulation of the distribution to check the actual data. Once this simulation is performed and the parameters meet the desired criteria, parametric tests can be performed to evaluate the data.

APPENDIX A

KRUSKAL-WALLIS TEST DATA

NOTE: The following tables provide the test data from the Kruskal-Wallis Test. The percents given are produced from the Cumulative Distribution Function which has values from 0 to 1.00.

DAYLIGHT TRIALS

	MAP A PCT	MAP B PCT	MAP C PCT	MAP D PCT	MAXIMUM DIFFERENCE
(0,5)	20	15	14	10	10
(0,10)	35	36	34	22	14
(0,15)	42	48	48	40	08
(0,20)	53	54	56	49	07
(0,25)	61	61	64	53	11
(0,30)	67	66	66	58	09
(0,35)	69	68	67	61	08
(0,40)	72	71	68	63	09
(0,45)	75	74	71	66	09
(0,50)	76	77	74	69	08
(0,55)	76	80	78	71	09
(0,60)	77	80	79	73	07
(0,65)	78	82	79	74	08
(0,70)	79	83	80	75	08
(0,75)	82	83	80	76	07
(0,80)	86	83	82	77	09
(0,85)	89	85	83	77	12
(0,90)	90	85	85	78	12
(0,95)	90	86	86	78	12
(0,100)	92	86	88	78	14

	MAP A PCT	MAP B PCT	MAP C PCT	MAP D PCT	MAXIMUM DIFFERENCE
(0,105)	92	86	89	78	14
(0,110)	92	86	90	81	11
(0,115)	92	86	90	83	09
(0,120)	94	89	90	84	10
(0,125)	94	89	90	85	09
(0,130)	94	91	90	85	09
(0,135)	94	92	90	85	09
(0,140)	94	93	90	88	06
(0,145)	95	93	90	90	05
(0,150)	95	93	90	90	05
(0,155)	95	94	90	91	05
(0,160)	95	94	90	93	05
(0,165)	96	94	91	93	05
(0,170)	96	94	91	93	05
(0,175)	96	94	92	93	04
(0,180)	96	95	92	93	04
(0,185)	96	96	92	94	04
(0,190)	96	98	92	94	06
(0,195)	96	98	93	94	05
(0,200)	96	98	93	94	05
(0,>200)	100	100	100	100	00

NIGHT TRIALS

	MAP A PCT	MAP B PCT	MAP C PCT	MAP D PCT	MAXIMUM DIFFERENCE
(0,5)	14	08	10	09	06
(0,10)	28	20	22	24	08
(0,15)	34	31	30	35	05
(0,20)	39	32	41	39	09
(0,25)	46	36	43	41	10
(0,30)	48	40	45	47	08
(0,35)	50	45	48	50	05
(0,40)	57	48	48	55	09
(0,45)	60	54	49	58	11
(0,50)	60	56	55	64	09
(0,55)	64	58	65	69	11
(0,60)	67	63	69	73	10
(0,65)	70	66	70	75	09
(0,70)	75	67	73	77	10
(0,75)	77	71	75	77	06
(0,80)	78	75	76	80	05
(0,85)	79	76	77	80	04
(0,90)	82	78	80	80	04
(0,95)	82	79	83	81	04
(0,100)	84	85	85	81	04

	MAP A PCT	MAP B PCT	MAP C PCT	MAP D PCT	MAXIMUM DIFFERENCE
(0,105)	86	89	89	82	07
(0,110)	86	89	89	82	07
(0,115)	86	90	89	83	07
(0,120)	86	90	89	85	05
(0,125)	88	91	90	85	06
(0,130)	88	91	91	85	06
(0,135)	88	91	91	85	06
(0,140)	88	93	91	86	07
(0,145)	89	95	92	88	07
(0,150)	89	95	92	88	07
(0,155)	90	96	94	88	08
(0,160)	91	96	94	88	08
(0,165)	91	98	96	89	09
(0,170)	91	98	96	89	09
(0,175)	91	98	96	89	09
(0,180)	91	99	96	89	10
(0,185)	92	99	96	90	09
(0,190)	93	100	97	92	08
(0,195)	95	100	97	93	07
(0,200)	96	100	97	93	07
(0,>200)	100	100	100	100	00

APPENDIX B

PARAMETERS GENERATED FOR THE EXPONENTIAL DISTRIBUTION

NOTE: The results given are from one thousand replications of 96 Exponential generated random variates. All values except for skewness, kurtosis and coefficient of variation are in meters. The skewness, kurtosis and coefficient of variation are dimensionless quantities.

A. RESULTS FOR DAYLIGHT NAVIGATION

1. Generated Means, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	42.23	44.10	47.40	47.22
Standard Deviation	4.233	4.637	4.938	4.651
Skewness	0.017	0.294	0.239	0.035
Kurtosis	0.131	0.526	0.367	-0.16
Minimum	29.13	31.00	31.02	32.40
Maximum	57.12	65.47	71.22	59.27

2. Generated Standard Deviation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	41.91	43.71	46.87	46.51
Standard Deviation	5.814	6.441	6.666	6.537
Skewness	0.437	0.504	0.563	0.420
Kurtosis	0.369	0.678	0.890	0.120
Minimum	24.68	27.32	28.40	30.03
Maximum	64.03	73.30	75.99	71.92

3. Generated Skewness, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	1.823	1.809	1.782	1.787
Standard Deviation	0.558	0.572	0.506	0.534
Skewness	1.088	1.316	1.063	1.004
Kurtosis	1.958	2.783	2.270	1.682
Minimum	0.727	0.632	0.728	0.717
Maximum	4.752	4.822	4.334	4.654

4. Generated Kurtosis, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	4.257	4.237	3.976	4.050
Standard Deviation	3.732	3.949	3.278	3.482
Skewness	2.220	2.485	2.259	1.963
Kurtosis	7.771	9.171	8.565	5.742
Minimum	-0.42	-0.81	-0.42	-0.60
Maximum	32.27	33.03	27.13	26.82

5. <u>Generated Coefficient of Variation, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	0.993	0.989	0.988	0.985
Standard Deviation	0.101	0.098	0.095	0.095
Skewness	0.629	0.467	0.481	0.546
Kurtosis	0.721	0.532	0.393	1.132
Minimum	0.752	0.689	0.714	0.731
Maximum	1.444	1.410	1.345	1.486

6. <u>Generated 50th Quantile, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	29.55	30.96	33.24	33.12
Standard Deviation	4.376	4.420	4.902	4.751
Skewness	0.299	0.257	0.291	0.313
Kurtosis	0.102	0.374	0.084	0.172
Minimum	18.18	18.07	20.20	20.55
Maximum	44.07	47.76	54.09	50.29

7. Generated 75th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	58.58	61.70	66.03	65.74
Standard Deviation	7.563	7.941	8.254	8.081
Skewness	0.287	0.312	0.240	0.196
Kurtosis	0.308	0.312	0.207	0.064
Minimum	37.21	41.37	40.31	42.60
Maximum	88.21	93.81	102.0	94.57

8. Generated 90th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	93.20	97.74	105.1	104.4
Standard Deviation	12.03	13.28	14.09	12.99
Skewness	0.291	0.369	0.392	0.255
Kurtosis	-0.04	-0.09	0.616	0.290
Minimum	61.97	67.00	68.06	62.43
Maximum	133.7	142.2	181.2	156.0

9. Generated Range, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	217.2	227.1	241.7	239.9
Standard Deviation	53.66	58.11	57.54	59.55
Skewness	1.002	1.168	1.066	0.974
Kurtosis	1.384	2.185	2.215	1.462
Minimum	98.69	117.5	120.7	128.5
Maximum	470.0	518.2	527.7	544.2

E. RESULTS FOR NIGHT NAVIGATION

1. Generated Means, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	46.42	62.98	54.79	63.07
Standard Deviation	4.653	6.612	5.708	6.212
Skewness	0.071	0.294	0.239	0.035
Kurtosis	0.131	0.526	0.367	-0.16
Minimum	32.02	44.21	35.86	43.27
Maximum	62.77	93.35	82.33	79.15

2. Generated Standard Deviation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	46.06	62.33	54.18	62.12
Standard Deviation	6.391	9.184	7.706	8.731
Skewness	0.437	0.504	0.563	0.420
Kurtosis	0.369	0.678	0.890	0.120
Minimum	27.13	38.96	32.83	40.11
Maximum	70.38	104.5	87.85	96.05

3. <u>Generated Skewness, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	1.823	1.809	1.782	1.787
Standard Deviation	0.558	0.572	0.506	0.534
Skewness	1.088	1.316	1.063	1.000
Kurtosis	1.958	2.783	2.270	1.682
Minimum	0.727	0.632	0.728	0.717
Maximum	4.752	4.822	4.334	4.654

4. <u>Generated Kurtosis, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	4.257	4.237	3.976	4.050
Standard Deviation	3.732	3.949	3.278	3.482
Skewness	2.220	2.485	2.259	1.963
Kurtosis	7.771	9.171	8.565	5.742
Minimum	-0.42	-0.81	-0.42	-0.59
Maximum	32.27	33.03	27.13	26.82

5. Generated Coefficient of Variation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	0.993	0.989	0.989	0.985
Standard Deviation	0.100	0.098	0.095	0.095
Skewness	0.629	0.467	0.481	0.546
Kurtosis	0.721	0.532	0.393	1.132
Minimum	0.752	0.689	0.714	0.731
Maximum	1.444	1.410	1.345	1.486

6. Generated 50th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	32.48	44.15	38.42	44.23
Standard Deviation	4.810	6.302	5.667	6.346
Skewness	0.299	0.310	0.291	0.313
Kurtosis	0.102	0.115	0.084	0.172
Minimum	19.98	20.05	23.35	27.45
Maximum	48.44	49.61	62.53	67.17

7. __Generated 75th Quantile, 1000 Replications__

	MAP A	MAP B	MAP C	MAP D
Mean	64.39	87.98	76.33	87.61
Standard Deviation	8.313	11.32	9.542	10.79
Skewness	0.287	0.312	0.240	0.196
Kurtosis	0.308	0.312	0.207	0.064
Minimum	40.90	58.99	46.60	57.17
Maximum	96.96	133.8	117.9	126.3

8. __Generated 90th Quantile, 1000 Replications__

	MAP A	MAP B	MAP C	MAP D
Mean	102.4	139.4	121.5	139.5
Standard Deviation	13.23	18.93	16.29	17.35
Skewness	0.291	0.369	0.392	0.255
Kurtosis	-0.04	-0.09	0.616	0.290
Minimum	68.12	95.54	78.67	83.38
Maximum	146.9	202.7	209.5	208.3

9. <u>Generated Range, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	238.7	323.8	379.4	320.5
Standard Deviation	58.98	82.85	66.41	79.53
Skewness	1.002	1.168	1.066	0.974
Kurtosis	1.384	2.185	2.215	1.462
Minimum	108.5	167.6	139.6	171.6
Maximum	516.6	738.9	610.0	726.8

APPENDIX C

PARAMETERS GENERATED FOR THE GAMMA DISTRIBUTION

NOTE: The results given are from one thousand replications of 96 Gamma generated random variates. All values except for skewness, kurtosis and coefficient of variation are in meters. The skewness, kurtosis and coefficient of variation are dimensionless quantities.

A. RESULTS FOR DAYLIGHT NAVIGATION

1. Generated Means, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	41.87	44.16	47.45	47.23
Standard Deviation	6.523	6.470	6.146	6.381
Skewness	0.251	0.460	0.183	0.350
Kurtosis	0.035	0.382	0.116	0.253
Minimum	23.13	26.65	28.52	28.84
Maximum	61.36	70.18	69.43	69.64

2. Generated Standard Deviation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	62.29	63.97	59.93	59.35
Standard Deviation	12.23	12.24	10.26	10.18
Skewness	0.452	0.657	0.596	0.504
Kurtosis	0.176	0.844	0.703	0.556
Minimum	27.92	33.46	35.33	34.72
Maximum	107.2	117.2	107.6	104.7

3. Generated Skewness, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	2.632	2.579	2.216	2.256
Standard Deviation	0.782	0.806	0.618	0.656
Skewness	1.196	1.374	1.116	1.017
Kurtosis	2.009	2.398	1.899	1.573
Minimum	1.170	0.983	0.876	1.012
Maximum	6.426	6.344	4.833	5.608

4. Generated Kurtosis, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	8.727	8.538	6.106	6.438
Standard Deviation	6.740	6.998	4.747	5.027
Skewness	2.112	2.156	2.115	1.920
Kurtosis	5.994	5.821	6.183	5.398
Minimum	0.379	-0.26	-0.42	-0.05
Maximum	50.22	49.38	33.25	39.62

5. Generated Coefficient of Variation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	1.488	1.448	1.262	1.256
Standard Deviation	0.175	0.178	0.129	0.131
Skewness	0.594	0.833	0.407	0.417
Kurtosis	0.590	1.060	0.087	0.604
Minimum	1.064	1.072	0.939	0.881
Maximum	2.139	2.165	1.754	1.926

6. Generated 50th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	17.24	19.12	25.42	25.67
Standard Deviation	4.432	4.795	5.193	5.155
Skewness	0.718	0.576	0.365	0.466
Kurtosis	0.880	0.467	0.001	0.555
Minimum	7.856	7.835	12.60	11.78
Maximum	39.58	38.74	45.40	46.97

7. Generated 75th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	54.23	58.09	64.76	64.54
Standard Deviation	10.98	10.99	10.18	10.66
Skewness	0.567	0.500	0.187	0.483
Kurtosis	0.499	0.337	-0.12	0.486
Minimum	29.66	29.46	34.73	37.29
Maximum	100.6	98.50	94.41	109.5

8. Generated 90th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	110.7	115.5	118.4	116.7
Standard Deviation	21.44	20.86	19.54	19.60
Skewness	0.356	0.555	0.484	0.439
Kurtosis	0.561	0.457	0.598	0.574
Minimum	54.49	67.09	66.96	66.83
Maximum	184.6	208.7	200.1	192.8

9. <u>Generated Range, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	347.7	357.7	318.2	318.4
Standard Deviation	109.2	116.5	87.98	88.99
Skewness	1.169	1.306	1.110	1.021
Kurtosis	2.233	2.682	1.831	1.797
Minimum	122.4	139.7	153.2	148.7
Maximum	908.6	948.3	732.1	821.4

E. RESULTS FOR NIGHT NAVIGATION

1. Generated Means, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	46.33	62.97	55.15	40.20
Standard Deviation	5.944	7.138	7.427	3.982
Skewness	0.164	0.201	0.219	0.159
Kurtosis	0.029	-0.02	0.188	-0.02
Minimum	28.18	42.92	34.40	28.46
Maximum	68.54	89.24	84.52	52.91

2. Generated Standard Deviation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	59.65	68.62	73.90	37.92
Standard Deviation	10.18	10.47	13.04	5.153
Skewness	0.555	0.499	0.641	0.420
Kurtosis	0.662	0.307	0.811	0.229
Minimum	36.17	42.32	40.87	25.64
Maximum	111.8	106.3	127.4	57.79

3. <u>Generated Skewness, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	2.322	1.979	2.352	1.698
Standard Deviation	0.689	0.627	0.699	0.507
Skewness	1.235	1.426	1.275	1.004
Kurtosis	2.024	3.485	2.708	1.772
Minimum	1.071	0.867	0.946	0.606
Maximum	5.465	5.201	6.611	4.127

4. <u>Generated Kurtosis, 1000 Replications</u>

	MAP A	MAP B	MAP C	MAP D
Mean	6.860	5.074	6.932	3.652
Standard Deviation	5.530	4.629	5.570	3.214
Skewness	2.115	2.670	2.273	2.136
Kurtosis	5.707	10.34	8.104	7.146
Minimum	0.097	-0.20	-0.13	-0.83
Maximum	37.64	36.24	52.32	25.65

5. Generated Coefficient of Variation, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	1.287	1.088	1.340	0.944
Standard Deviation	0.141	0.114	0.151	0.090
Skewness	0.581	0.672	0.687	0.434
Kurtosis	0.304	1.040	1.055	0.554
Minimum	0.942	0.814	0.945	0.728
Maximum	1.843	1.598	2.063	1.328

6. Generated 50th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	24.50	40.62	27.01	29.14
Standard Deviation	4.929	6.758	5.950	4.069
Skewness	0.544	0.296	0.449	0.259
Kurtosis	1.010	0.233	0.220	0.102
Minimum	11.96	19.01	10.51	18.09
Maximum	50.31	68.74	49.99	42.18

7. Generated 75th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	62.78	87.77	74.26	56.11
Standard Deviation	10.37	12.12	12.76	6.752
Skewness	0.413	0.188	0.244	0.207
Kurtosis	0.280	-0.16	-0.15	0.023
Minimum	36.81	52.85	38.54	35.18
Maximum	110$7	126.9	113.9	80.57

8. Generated 90th Quantile, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	115.6	145.9	141.0	87.18
Standard Deviation	18.63	21.03	24.61	11.22
Skewness	0.303	0.429	0.435	0.326
Kurtosis	0.071	0.335	0.463	0.369
Minimum	68.04	90.24	80.58	54.44
Maximum	183.6	234.3	248.6	126.1

9. Generated Range, 1000 Replications

	MAP A	MAP B	MAP C	MAP D
Mean	322.2	360.4	397.8	194.2
Standard Deviation	92.13	96.13	117.6	44.55
Skewness	1.288	1.278	1.316	0.895
Kurtosis	2.910	2.937	2.728	1.047
Minimum	153.5	185.9	190.2	95.04
Maximum	798.5	889.9	1059.1	396.9

LIST OF REFERENCES

1. DCS, Experimetntation, U.S. Army Combat Developments Experimentation Command and BDM, Scientific Support Laboratory, Test of New and Improved Maps and Map Products (Phase III), Final Report, p. 361, , 1977.

2. A. S. Goodman/P. A. W. Lewis/H. e. Robbins, Simultaneous Estimation of Large Numbers of Extreme Quantiles in Simulation Experiments, p. 31, , 1971.

3. George S. Fishman, Concepts and Methods in Discrete Event Simulation, p. 399, , 1973.

4. Leo Breiman, Statistics, With a View Toward Applications, p. 413, , 1973.

5. michael Woodroofe, Probability with Applications, p. 379, Robert H. Summersquill and Shelly Levine Langman, 1975.

6. Oscar Kempthorne and Leroy Folks, Probability, Statistics and Data Analysis, p. 555, , 1971.

7. Vasant S. Huzurbazar, Sufficient Statistics, p. 270, Anant M. Kshirsagar, 1976.

8. D. A. S. Fraser, Nonparametric Methods in Statistics, p. 299, , 1957.

9. Programs Developed at the Health Sciences Compution Facility, UCLA, Biomedical Computer Programs, p. 890, W. J. Dixon, 1975.

10. Harold J. Larson, Introduction to Probability Theory and Statistical Inferrence, p. 430, , 1969.

11. John Neter and William Wasserman, *Applied linear Statistical Models*, p. 842, , 1976.

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